Extensive investigation of pion-pion scattering in the quark potential model

Guo-Quan Zhao, Xiao-Gong Jing, and Jun-Chen Su
Center for Theoretical Physics, Department of Physics, Jilin University, Changchun 130023, People's Republic of China
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The QCD-inspired constituent quark potential model is employed to derive the pion-pion interacting potential by means of the resonating group method. In the derivation, we start from the t-channel one-gluon exchange potential and the s-channel one-gluon exchange potential we derived previously in the approximation of order v^2/c^2. Use of the derived pionic potential to calculate low-energy pion-pion elastic scattering phase shifts leads to results which are in reasonably good agreement with the present experimental data. [S0556-2821(98)05817-2]

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Motivated by the success achieved in studies of hadron spectroscopy based on the QCD-inspired constituent quark potential model [1,2], there have been continued efforts to derive hadron-hadron interactions at the quark-gluon level. The first effort was concentrated on the nucleon-nucleon (NN) interaction and led to an understanding that the short-range repulsive core arises from the quark and gluon exchanges [3–5], and even for the intermediate-range attractive force, it is likely to be explained as a spatial distortion of the three-quark clusters [6]. Subsequently, the nonrelativistic quark potential model and the resonating group method [7], used widely in the study of NN interactions, were applied to investigations of the K^+N interaction [8] and πN interaction [9]. The results given in these investigations are, qualitatively, compatible with experiments. In recent years, the study of meson-meson interactions has attracted more interest. In this study, Weinstein and Isgur proposed a variational approach based on the quark potential model [10]. Using this approach, they obtained an effective potential between two pseudoscalar mesons and found weakly bound $\bar{K}K$ molecules. Meanwhile, Barnes and Swanson developed a Born order diagrammatic technique within the quark potential model to evaluate the interaction potentials and elastic scattering phase shifts for some special meson systems such as $\pi\pi$, πK, etc. The results they obtained seem to be in remarkably good agreement with the experiment [11,12].

The quark potential model used in the aforementioned works, usually, is composed of one-gluon exchange potentials (OGEPs) plus a confining potential. If we are limited to study interacting systems such as NN, $\pi^+\pi^+$, K^+K^+, π^+K^+, and similar others, because no quark-antiquark pair ($q\bar{q}$) annihilation appears, it is only necessary to start from the t-channel OGEP. However, for studying other interacting processes of the systems such as πN, $\pi^+\pi^-$, and $K\bar{K}$ in which the $q\bar{q}$ annihilation is involved, the s-channel OGEP is necessary to be incorporated in the quark potential model. In our previous work [13], a complete expression of the s-channel OGEP was derived in the approximation of order v^2/c^2. This OGEP may yield attractive hadronic potentials when the constituent quark masses take values commonly used in hadron spectroscopy. In this paper, we confine ourselves to investigate the $\pi\pi$ interaction and low-energy elastic scattering based on the quark potential model in which the one-gluon exchange potentials are those we derived. Ordinarily, the $\pi\pi$ scattering is viewed as a complicated process. Apart from the t-channel and s-channel gluon exchanges, the t-channel resonance exchange and s-channel resonance production probably also contribute to the process. In this paper, we do not consider the latter two mechanisms. The reason is apparent. The t-channel resonance exchange such as pion exchange, we think, would not be present in $\pi\pi$ scattering. The s-channel resonance, according to the quark model, is formed by the underlying interaction which is already included in the model used. Therefore, if the contribution from the intermediate resonance production is also taken into account, we would face the double counting problem. As one knows, the microscopic mechanisms of the $\pi\pi$ interaction for different isospin channels are different. In the $I=2$ channel, as discussed in Ref. [11], the t-channel gluon exchange dominates only, while for $I=0,1$ channels, the s-channel one-gluon exchange takes part in the interaction and gives an even more important contribution to the scattering process than the t-channel one-gluon exchange. In this paper, we will pay attention mainly to the scattering in the $I=0,1$ channels which have not been discussed within the potential model in the previous literature. First, we derive the pionic potential from the underlying quark and/or antiquark interactions by means of the resonating group method. In this method, the two-pion system is treated as two $q\bar{q}$ clusters. Each cluster is a color singlet and of spin zero and isospin one. The wave function of the system is usually written, in momentum space, in the form

$$\psi=\int d^3\rho\,\Phi(\vec{p}_1,\vec{p}_2,\vec{p}_3,\vec{p}_4;\vec{\rho})f(\vec{\rho}), \tag{1}$$

where $f(\vec{\rho})$ is a function describing the relative motion of the two $q\bar{q}$ clusters and $\Phi(\vec{p}_1,\ldots,\vec{p}_4;\vec{\rho})$ is the basis wave function for the four-particle system. The latter function is chosen to be

$$\Phi(\vec{p}_1,\ldots,\vec{p}_4;\vec{\rho})=A\chi^{SIC}(1,2,3,4)R(\vec{p}_1,\ldots,\vec{p}_4;\vec{\rho}), \tag{2}$$

where $\chi^{SIC}(1,2,3,4)$ stands for the spin, isospin, and color wave functions, $R(\vec{p}_1,\ldots,\vec{p}_4;\vec{\rho})$ the spatial wave function constructed from the lowest-lying harmonic oscillator states of the four particles, and A symbolizes the antisymmetrization operator $A=1-P_{13}-P_{24}+P_{13}P_{24}$; here, P_{ij} are the interchange operators. This operator simultaneously ensures the total wave function to be symmetric under the interchange of two pions.

The interaction potential for the four-particle system is given by

$$V=\sum_{i<j=1}^{4} V_{ij}. \tag{3}$$

In our model, V_{ij} is taken to be

$$V_{ij}=V_{ij}^{t}+V_{ij}^{s}+V_{ij}^{c}, \tag{4}$$

where V_{ij}^t, V_{ij}^s, and V_{ij}^c, respectively, stand for the t-channel OGEP, the s-channel OGEP, and the confining potential. The t-channel OGEP is chosen to be [13]

$$V_{ij}^{t}=\frac{4\pi\alpha_s\hat{C}_{ij}^{t}}{|\vec{q}-\vec{k}|^2}\Bigg\{1-\frac{\vec{P}^2}{4m^2}+\frac{\vec{q}\cdot\vec{k}}{m^2}+\frac{i}{8m^2}[\vec{p}\times(\vec{q}-\vec{k})]\cdot(\vec{\sigma}_i-\vec{\sigma}_j)+\frac{3i}{4m^2}(\vec{q}\times\vec{k})\cdot(\vec{\sigma}_i+\vec{\sigma}_j)$$
$$-\frac{1}{6m^2}(\vec{q}-\vec{k})^2\vec{\sigma}_i\cdot\vec{\sigma}_j+\frac{1}{4m^2}(\vec{q}-\vec{k})^2T_{ij}(\vec{q}-\vec{k})\Bigg\}, \tag{5}$$

where

$$\hat{C}_{ij}^{t}=\begin{cases}\frac{\lambda_i^a}{2}\frac{\lambda_j^a}{2}\left(\frac{\lambda^{a*}}{2}\frac{\lambda^{a*}}{2}\right), & \text{for } qq(\overline{qq}),\\ -\frac{\lambda_j^a}{2}\frac{\lambda_i^{a*}}{2}, & \text{for } q\overline{q},\end{cases} \tag{6}$$

is the color matrix, $\vec{\sigma}_i$ is the ith-particle spin Pauli matrix, $T_{ij}(\vec{q}-\vec{k})$ the tensor force operator, and $\vec{P}$, $\vec{k}$, and $\vec{q}$ the total momentum and the relative momenta of the initial and final states, respectively. The s-channel OGEP is [13]

$$V_{ij}^{s}=\frac{\pi\alpha_s}{2m^2}\hat{F}_{ij}^{s}\hat{C}_{ij}^{s}\Bigg\{3+\vec{\sigma}_i\cdot\vec{\sigma}_j-\frac{1}{4m^2}[2\vec{P}^2+8(\vec{q}^2+\vec{k}^2)]-\frac{1}{4m^2}[\vec{P}^2+4(\vec{q}^2+\vec{k}^2)]\vec{\sigma}_i\cdot\vec{\sigma}_j+\frac{1}{4m^2}[\vec{P}\times(\vec{q}-\vec{k})]\cdot(\vec{\sigma}_i-\vec{\sigma}_j)$$
$$+\frac{1}{4m^2}[\vec{P}\cdot\vec{\sigma}_i(\vec{q}+\vec{k})\cdot\vec{\sigma}_j-(\vec{q}+\vec{k})\cdot\vec{\sigma}_i\vec{P}\cdot\vec{\sigma}_j+4\vec{k}\cdot\vec{\sigma}_j\cdot\vec{q}\cdot\vec{\sigma}_j+4\vec{q}\cdot\vec{\sigma}_i\cdot\vec{q}\cdot\vec{\sigma}_j]\Bigg\}, \tag{7}$$

where $\hat{C}_{ij}^s$ and $\hat{F}_{ij}^s$ are the s-channel color matrix and the isospin operator, respectively:

$$\hat{C}_{ij}^{s}=\frac{1}{24}(\lambda_i^a-\lambda_j^{a*})^2,\quad \hat{F}_{ij}^{s}=\frac{1}{2}(1-\vec{\tau}_i\cdot\vec{\tau}_j). \tag{8}$$

Here $\vec{\tau}_i$ is the isospin Pauli matrix of the ith particle. The confining potential may be chosen to be a linear one, a harmonic oscillator one, or some other. For convenience, we simply take the harmonic oscillator potential which is expressed in momentum space as

$$V_{ij}^{c}=\hat{C}_{ij}^{t}(2\pi)^3\mu\omega^2\nabla_k^2\delta^3(\vec{q}-\vec{k}), \tag{9}$$

where μ is the reduced mass of the two interacting particles and ω the force parameter. With the wave function and the potential given above, according to the well-known procedure [7,8], we can derive the resonating group equation (RGE) satisfied by the function $f(\vec{\rho})$ from the Schrödinger equation satisfied by the system of two quarks and two antiquarks and then convert the RGE to a Schrödinger-type equation. In the Schrödinger equation, the pionic potential generally is nonlocal and in our model it may be written as

$$V(\vec{R},\vec{R}')=V_s^{D}(\vec{R},\vec{R}')+V_s^{ex}(\vec{R},\vec{R}')+V_t^{ex}(\vec{R},\vec{R}')$$
$$+V_c^{ex}(\vec{R},\vec{R}')+T^{ex}(\vec{R},\vec{R}')+EN^{ex}(\vec{R},\vec{R}'), \tag{10}$$

where the first term is called the direct term and the other terms are exchange terms. From the color matrix elements, one can see that the direct term only arises from the s-channel OGEP because the color matrix elements of the other potentials in Eq. (4) are zero. But all the potentials in Eq. (4) give contributions to the exchange term. The five exchange terms in Eq. (10) are, respectively, given by the s-channel OGEP, the t-channel OGEP, the confining potential, the kinetic energy, and the normalization function ap-

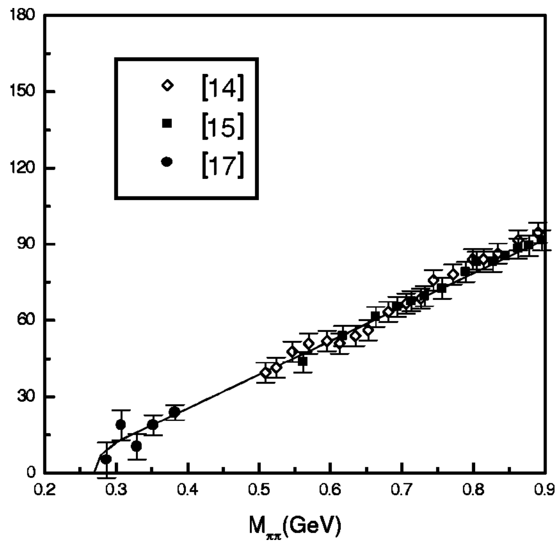

FIG. 1. The theoretical and experimental phase shifts for the $\pi\pi I=0$ channel S-wave scattering.

pearing in the RGE. In our calculation, these potentials are given exact and explicit expressions which are used to compute the transition amplitude.

In order to test whether the potentials derived give a good description of the interaction between two pions or not, we calculate phase shifts of the low-energy $\pi\pi$ scattering and compare the theoretical results with experiments. The calculation will be carried out in the Born approximation. As was demonstrated in Ref. [11], the Born order approximation can reasonably describe hadron elastic scattering processes. In this approximation and center of mass frame, the transition amplitude of two-pion scattering is expressed as

$$T^I_{ji}(\mu)=\int d^3Rd^3R'e^{-i\vec{k}\cdot\vec{R}}V(\vec{R},\vec{R}')e^{i\vec{k}'\cdot\vec{R}'}. \tag{11}$$

The lth partial wave phase shift evaluated by this amplitude is represented as

$$\delta^I_1=-\frac{Mk}{16\pi}\int_{-1}^{1}d\mu P_1(\mu)T^I_{ji}(\mu), \tag{12}$$

where $\mu=\cos\theta$, θ is the scattering angle, M is the total energy of the two-pion system, k is the relative momentum of the two pions, $P_1(\mu)$ is the Legendre polynomial of lth rank, and I designates the isospin. In our calculation, the QCD coupling constant, α_s, the quark mass, m_q, the size parameter of the harmonic oscillator, b, and the force strength of the confining potential, ω, are chosen to be $\alpha_s=0.4$, $m_q=0.33$ GeV, $b=0.286$ fm, and $\omega=0.8$ GeV, which are consistent with the values commonly used in the quark potential model. The calculated phase shifts as well as the existing experimental data [14–18] are displayed in Figs. 1–4. Figures 1 and 2 show a remarkably good fit of the theoretical results to the experimental data in the range from threshold to about 1 GeV for the $I=0,2$ channel S-wave phase shifts. In Fig. 3, the theoretical phase shifts for the $I=2$ channel D-wave scattering are qualitatively in good agreement with

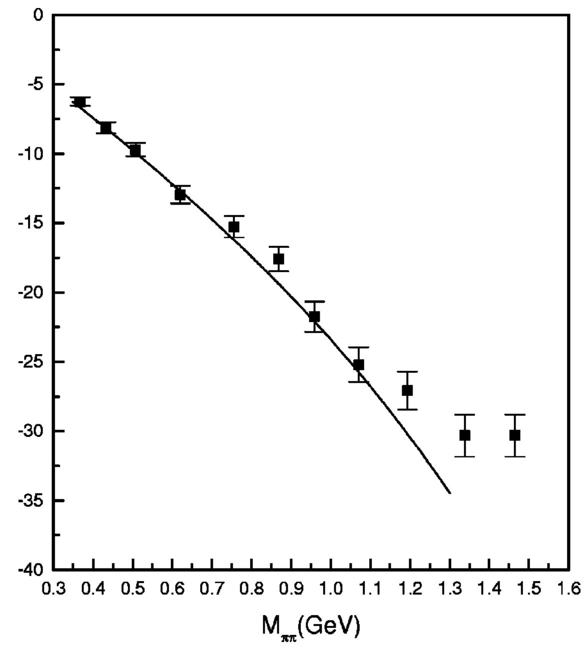

FIG. 2. The theoretical and experimental phase shifts for the $\pi\pi I=2$ channel S-wave scattering.

the experimental ones, although the calculated result is smaller than the experimental one [14]. The difference between both is less than 1°. As for the other partial-wave phase shifts, the experimental data are rather poor and not so definite [14–18]; so our results represented in Fig. 4 can only be viewed as theoretical predictions which need to be verified by future experiments. From our calculations, it is found that the underlying mechanism of each partial-wave scattering strongly depends on its isospin state, and even in the same isospin channel the interaction origins for different partial-wave scattering processes are, in general, different from each other. In the $I=0$ channel, our calculation shows

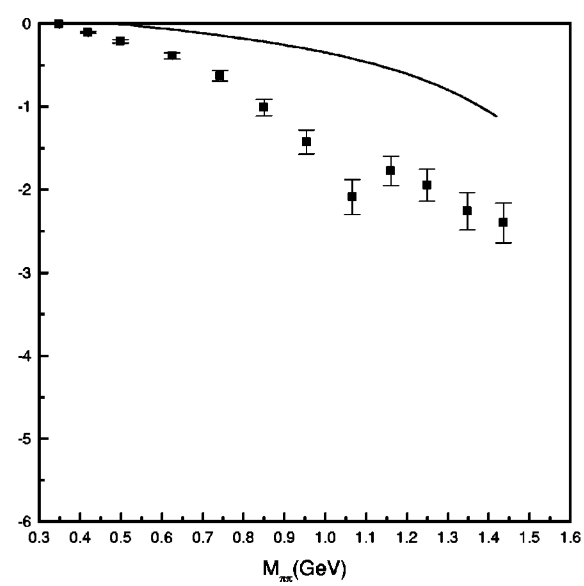

FIG. 3. The theoretical and experimental phase shifts for the $\pi\pi I=2$ channel D-wave scattering.

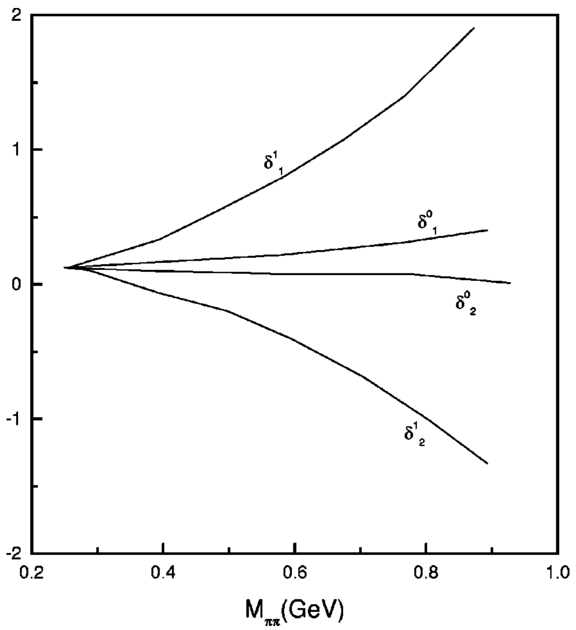

FIG. 4. The calculated phase shifts for the $I=0,1$ channel P-wave and D-wave scattering.

that the S-wave phase shift is almost equal to that derived from the s-channel OGEP since there is a partial cancellation among the phase shifts given by the last four potentials in Eq. (10). This fact indicates that the interaction arising from the $q\bar{q}$ annihilation and creation is dominant in the $I=0$ S-wave scattering and gives rise to an attractive pionic potential as seen from the positiveness of the phase shift given by the interaction. For the P-wave scattering, the phase shift, we find, is only governed by the s-channel OGEP, while the $I=0$ D-wave phase shift is merely given by the first three potentials in Eq. (10). For the $I=1$ channel scattering, since the isospin matrix element in the last four terms in Eq. (10) is zero when $I=1$ and there is a cancellation between the first two terms in Eq. (10) due to the fact that the isospin matrix elements in both potentials have opposite signs, the phase shifts are much smaller. That is why there are no experimental data to be found in the literature. For the $I=2$ channel scattering, the isospin matrix elements in the first two potentials in Eq. (10) vanish; therefore, the phase shifts in this channel are only determined by the interactions described by the last four terms in Eq. (10). Especially, only the potential $V_t^{ex}(\vec{R},\vec{R}')$ is responsible for the D-wave scattering.

Our calculated phase shifts show that the model used manifests itself to be quite successful in reproducing the experimental data in the low-energy regime although the model is simple, concerning only basic interactions for quarks and antiquarks. Particularly, the incorporation of the s-channel OGEP allows us in a consistent way to describe the $\pi\pi$ scattering for all isospin channels and gives a very good fit to the S-wave scattering. Of course, in a more elaborate investigation, the model may be improved by considering higher-order effects of QCD such as the two-gluon exchange potential and the renormalization correction as well as a more sophisticated confining potential. In addition, the reasonability of the model should be verified by more comprehensive investigations on the πK, $K\bar{K}$, πN, KN, and other hadron scattering, which will be reported in the near future.

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